Control of telerobots with variable communication delay

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Abstract

This paper proposes two solutions for the control of telerobots, in which master and slave are connected through a communication system that introduces a variable delay. This is the typical case of packet-switched networks (e.g. Internet), in which the delay varies in an unpredictible way. So far, only constant delay has been considered in the design of the controllers for telerobots, and the known techniques may fail in case of variable delay. We propose here two controllers that ensure stable behaviour of the telerobot in case of variable delay. The first one, based on a particular Lyapunov function, is designed for position-error-based force-feedback configuration, while the second, exploiting passivity and wave decomposition, is designed for direct force-feedback systems. Formal proofs are accompanied by preliminary experimental results obtained with an Internet-connected teleoperator.

1 Introduction

In several applications of telerobots, the communication system introduces a variable delay. This is the case of submarine robots, controlled from the sea surface level through a sonar-like communication system, in which the delay increases as the remote robot goes deeper. Also, space operations have a communication delay that varies with the distance between orbiting device and Earth. Recently, the use of ISDN and packet-switched communication systems has been proposed and demonstrated to connect a master system with a remote slave. In particular, Internet-based communication are now allowing millions of users to interact with each other by exchanging textual, audio and video information. A major breakthrough would be to make use of this obiquitous communication system in telerobotic applications. As proposed in [7], this would open the way to more affordable applications of teleoperated systems in several fields, such as health care [4], tele-medicine, collaborative design [3], enterteinment and teleconferencing. Several test has been conducted on Internet-based [12] and ISDN-based [10] teleoperators, showing that the communication delay can be characterized by a slowly varying average value plus a fast-varying, random part.

So far, the research on the control of teleoperators with communication delay has dealt only with constant delay [1, 11, 6]. Since it has been demonstrated that the variability of the delay constitutes an additional source of instability [9], then the design of the controller for a teleoperator must take into account this characteristic.

This paper addresses the problem of the stabilization of teleoperators with variable communication delay. The model of the teleoperator with variable communication delay is introduced in Section 2. For such a system, two typical configuration are considered in the following. In particular, Section 3 reports the design of a decentralized state feedback for a position-error-based force-feedback scheme, while the problem of the stabilization of a teleoperator with direct, i.e. sensor based, force feedback is addressed in Section 4. Section 5 presents an experimental result obtained with one of the proposed control schemes. Finally Section 6 summarizes the paper and presents our plans for future research in this area.

2 Teleoperator with variable communication delay

The complete model of a telerobot with variable communication delay is shown in Figure 1. Here, y_m and y_s are the variables exchanged between master and slave. It is worthnoticing that T_1 (t) is usually different from T_2 (t). If linearity is assumed, this model can be simplified by collapsing the two delays into a single one, arbitrarily located in the forward direction. Another simplification is to have two identical delays

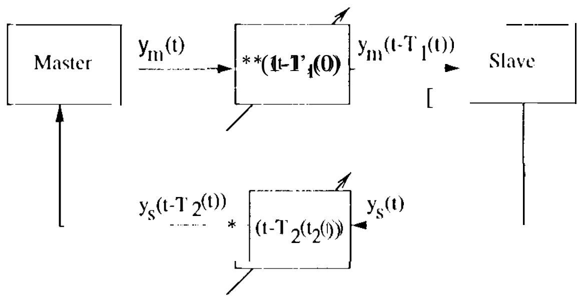

Figure 1: Telerobot with variable communication delay

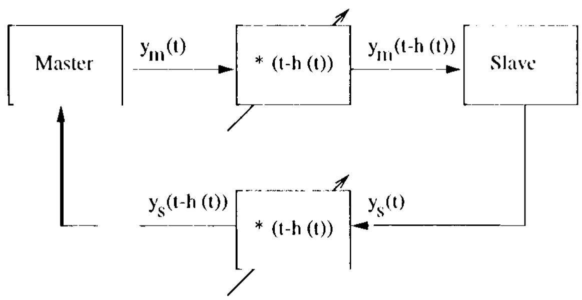

Figure 2: Simplified model of telerobot with variable communication delay.

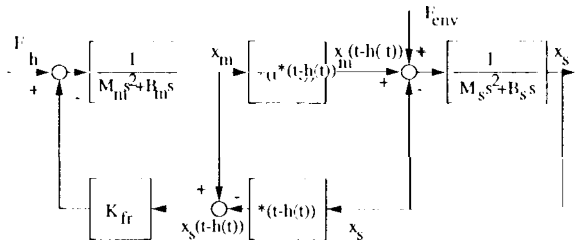

Figure 3: Position-based force feedback

in both directions, each of them with a value $h(t) = \frac{1}{2}(T_1(t) + T_2(t))$, as shown in Figure 2. From the point of view of the stability, in case of linearity, all the models are equivalent and the most suitable ones are used in the following sections.

As a further simplification of the problem, a single degree-of-freedom (dof) system is considered, since the results can be easily extended to multi dof systems.

3 Controller for position-error-based force-feedback telerobot

We consider the standard position-based force feedback scheme [5], and we propose a new decentralized controller based on state variables feedback. In this scheme, the forces acting on the master are proportional to the difference between the position of the master and that of the slave. For simplicity, we consider the single dof system shown in Figure 3 (with the communication delay modeled as in Figure 2), whose space-state equations are given by:

$$\Sigma_1 : \dot{x}_1 = A_1 x_1(t) + B_1 u_1(t) + A_{21} x_2(t - h(t)) \quad (1)$$

$$\Sigma_2 : \dot{x}_2 = A_2 x_2(t) + B_2 u_2(t) + A_{12} x_1(t - h(t)) \quad (2)$$

where x_1 and x_2 represents the full state of master and slave, respectively (i.e. $x_1 = [x_m, \dot{x}_m]^T$, $x_2 = [x_s, \dot{x}_s]^T$), u_1 and u_2 are the inputs to the sub-systems. The matrix coefficients of the state equations are given by:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_m}{M_m} \end{bmatrix}; \; B_1 = \begin{bmatrix} 0 \\ \frac{1}{M_m} \end{bmatrix};$$
$$A_{21} = \begin{bmatrix} 0 & 0 \\ \frac{K_{fr}}{M_m} & 0 \end{bmatrix}; \quad (3)$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_s}{M_s} \end{bmatrix}; \; B_2 = \begin{bmatrix} 0 \\ \frac{1}{M_s} \end{bmatrix};$$
$$A_{12} = \begin{bmatrix} 0 & 0 \\ \frac{K_p}{M_s} & 0 \end{bmatrix}; \quad (4)$$

where K_{fr} represents the force feedback gain, K_p represents the gain of the slave controller, and M_m, B_m, M_s, B_s represent masses and friction coefficients of the master and the slave.

For the system represented by equations (1) we propose the decentralized state feedback controller given by the following equations:

$$\begin{cases} u_1 = K_1 x_1 \\ u_2 = K_2 x_2 \end{cases} \quad (5)$$

where $K_1 = [K_1(1), K_1(2)]$ and $K_2 = [K_2(1), K_2(2)]$ are two gain vectors, as shown in Figure 4. Since state feedback ensures correct tracking only when reference and feedback are multiplied by the same gain, it follows that A_{21} and A_{12} depend on the controller gains:

$$A_{21} = \begin{bmatrix} 0 & 0 \\ \frac{K_1(1)}{M_m} & 0 \end{bmatrix}; \quad A_{21} = \begin{bmatrix} 0 & 0 \\ \frac{K_2(1)}{M_s} & 0 \end{bmatrix} \quad (6)$$

Note that this constraint makes solution methods such as the one described in [14] inapplicable to a position-based force reflecting telerobot.

The state equations (1) are rewritten in compact form including the controller equations (5) as:

$$\Sigma : \dot{x}(t) = A_k x(t) + A_d x(t - h(t)) \quad (7)$$

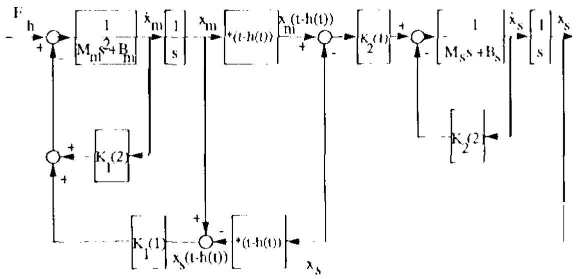

Figure 4: Block diagram of the state feedback controller.

where Σ represents the overall system, and the matrix coefficients are:

$$A_k = \begin{bmatrix} A_1 - B_1K_1 & 0 \\ 0 & A_2 - B_2K_2 \end{bmatrix};$$

$$A_d = \begin{bmatrix} 0 & A_{21} \\ A_{12} & 0 \end{bmatrix}; \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad (8)$$

The design criteria for the decentralized feedback are found using the following Lyapunov functional:

$$V(x,t) = x(t)^T x(t) + \frac{1}{1-\bar{\tau}} \int_{t-h(t)}^{t} x(\theta)^T x(\theta) d\theta \geq 0 \qquad (9)$$

where we have:

$$\dot{h}(t) \leq \bar{\tau} \leq 1 \quad \forall t \qquad (10)$$

The differentiation of equation (9) along the solution of (7) yields:

$$\dot{V}(x,t) = 2x^T(A_k x + A_d x_h) + \frac{1}{1-\bar{\tau}} x^T x + \frac{1-\dot{h}(t)}{1-\bar{\tau}} x_h^T x_h \qquad (11)$$

where, to improve readability, we set $x = x(t)$ and $x_h = x(t - h(t))$.

By using inequality

$$2x^T A_d x_h \leq x^T A_d A_d^T x + x_h^T x_h \qquad (12)$$

equation (11) becomes

$$\dot{V}(x,t) \leq x^T[(2A_k + A_d A_d^T) + \frac{1}{1-\bar{\tau}} I_n]x + x_h^T[I_n - \frac{1-\dot{h}(t)}{1-\bar{\tau}} I_n]x_h \qquad (13)$$

where I_n is a rank-n identity matrix.

The asymptotic stability of (7) is guaranteed if $\dot{V}(x, t) < 0$. This inequality holds if the matrices in the two quadratic forms of the right side of (13) are both negative-definite. Given equation (10), the controller must satisfy:

$$S = 2A_k + A_d A_d^T + \frac{1}{1-\bar{\tau}} I_n < 0 \qquad (14)$$

Condition (14) holds if all the eigenvalues of the matrix S are in $Re[s] < 0$, i.e.

$$Re\lambda_i(S) < 0 \quad \forall i \qquad (15)$$

Matrix S can be re-written in block form as:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad (16)$$

where:

$$S_{11} = \begin{bmatrix} \alpha & 2 \\ -2(\frac{K_1(1)}{M_m}) & -2(\frac{K_1(2)+B_m}{M_m}) + (\frac{K_1(1)}{M_m})^2 + \alpha \end{bmatrix}$$

$$S_{22} = \begin{bmatrix} \alpha & 2 \\ -2(\frac{K_2(1)}{M_s}) & -2(\frac{K_2(2)+B_s}{M_s}) + (\frac{K_2(1)}{M_s})^2 + \alpha \end{bmatrix}$$

$$S_{12} = S_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (17)$$

and $\alpha = \frac{1}{1-\bar{\tau}}$.

From equation (16) it follows that matrix S has a block diagonal structure and, therefore, the eigenvalues of S are the eigenvalues of each block. Since both blocks have the same structure and each gain only appears in the block relative to its controller, the solution of equation (15) is significantly simplified, and it is possible to design independent controllers for master and slave.

Let's now consider the upper-left block of (16). It can be rewritten as follows:

$$S_{11} = \begin{bmatrix} \alpha & 2 \\ -\xi_{11} & -\xi_{21} + \alpha \end{bmatrix}; \quad \xi_{11} = \frac{2K_1(1)}{M_m};$$

$$\xi_{21} = 2(\frac{K_1(2) + B_m}{M_m}) - (\frac{K_1(1)}{M_m})^2 \qquad (18)$$

The eigenvalues of S_{11} are computed using its characteristic polynomial:

$$det(sI_2 - S_{11}) = s^2 + s(\xi_{21} - 2\alpha) + 2\xi_{11} + \alpha^2 - \xi_{21}\alpha \qquad (19)$$

The equation (19) has roots with negative real part if the equation coefficients are positive, i.e.

$$Re(\lambda_i(S_1)) < 0 \Leftrightarrow \begin{cases} 2\xi_{11} + \alpha^2 - \xi_{21}\alpha & > 0 \\ \xi_{21} - 2\alpha & > 0 \end{cases} \qquad (20)$$

The following stability conditions are then derived:

$$\left\{ \begin{array}{lcl} \xi_{21} & = & \gamma\alpha \\ \xi_{11} & > & \frac{\gamma-1}{2}\alpha^2 \end{array} \right. ; \quad \gamma > 2 \tag{21}$$

where γ is a free design parameter that influences the performance of the teleoperator.

Eventually, from (21) we obtain the values for the feedback gains to be used in the Controller (5).

$$K_1(1) > \frac{\gamma-1}{4} M_m \alpha^2 \tag{22}$$

$$K_1(2) > \gamma\alpha M_m + \frac{K_1(1)^2}{M_m} - B_m \tag{23}$$

The equation (22) and (23) have always a solution. The same design procedure can be applied to the slave controller, then, given a teleoperator with position error based force feedback and variable communication delay, it is always possible to find a decentralized controller (5) that stabilizes the system.

A first remark on the result obtained is that it guarantees the stability of the two subsystems (master and slave) even in case of interruption on the communication link. In this case, we have $A_d = 0$ in (7). The stability of such a system is guaranteed in $A_k < 0$, which is guaranteed by (13). Another remark is that even if the proposed solution has been developed for a telerobot with a position scale factor equal to 1, it is still valid with different scale factors. In fact, if K_s is the scale factor from the master to the slave, a block with gain $\frac{1}{K_s}$ must be placed on the return path from the slave to the master, in order to preserve consistency and effectiveness of the force feedback. Then it is easy to prove that stability is not affected if $K_s \neq 1$. Finally, it is worthnoticing that the stability depends on the value of the time derivative of the delay, but not on the value of the delay itself. This means that we have provided a stability that is independent of delay (I OD). This improves previous results ([6]), on the control of teleoperators with position-error-based force-feedback, which dealt with constant delays and provided a non-analytical tool for the design of a stabilyzing compensator.

4 Stabilization of force-feedback telerobots

In this section we will introduce a method that guarantees the stability of direct force-feedback with a variable communication delay. This is an extension of the method using passivity and wave scattering concepts presented in [11]. In that paper, it is shown that the

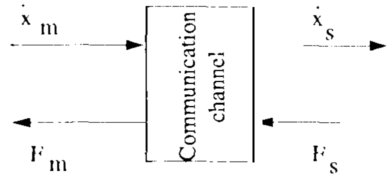

Figure 5: Communication system as a *2-port* system

communication system results passive if the so-called *wave variables* are sent instead of the real velocities and forces (the so-called *power variables*). The passivity of the communication system, in turn, guarantees the stability of the overall system. The communication channel for the telerobot is a typical *2-port* element, as shown in Figure 5, where $\dot{x}_m$ is the master velocity, F_m is the received force signal, $\dot{x}_s$ is the slave velocity set-point and F_s is the force generated during the interaction of the slave with the environment. It has been demonstrated that such system may not be passive, but an appropriate decomposition of the power variables into wave variables leads to a passive communication system. The result in [11] does not hold in case of variable communication delay, however the concept of wave scattering can still be exploited in order to get the overall passivity of the teleoperator.

Let us now recall some definitions. A system with input vector x and output vector y, it is said to be passive if its power flow satisfies the following equation:

$$P = \frac{dE}{dt} + P_{diss} \tag{24}$$

where E is a lower bounded energy function and P_{diss} is a non-negative power dissipation term.

When the *2-port* element of Figure 5 is considered, the power flow is:

$$P = \dot{x}_m F_m - \dot{x}_s F_s \tag{25}$$

By applying wave decomposition to the power variables, the power flow can be rewritten as follows:

$$P = \frac{1}{2}u_m^T u_m - \frac{1}{2}v_m^T v_m + \frac{1}{2}u_s^T u_s - \frac{1}{2}v_s^T v_s \tag{26}$$

We can notice that u_m and u_s increase the power flow into the system, while v_m and v_s decrease it. Then, the first two can be interpreted as *input variables*, while the other two can be interpreted as *output variables*.

Equation (26) implicitly defines the following wave variables:

$$\left\{ \begin{array}{lcl} u_m & = & \frac{1}{\sqrt{2b}}(b\dot{x}_m + F_m) \\ v_m & = & \frac{1}{\sqrt{2b}}(F_m - b\dot{x}_m) \end{array} \right. \tag{27}$$

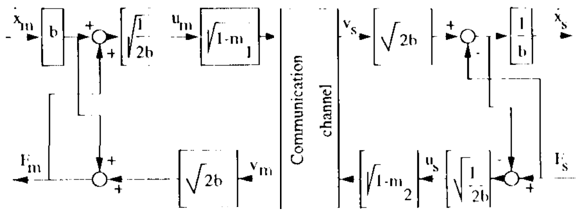

Figure 6: Wave transformations using master velocity and slave force

$$\begin{cases} u_s &= \frac{1}{\sqrt{2b}}(F_s - b\dot{x}_s) \\ v_s &= \frac{1}{\sqrt{2b}}(F_s + b\dot{x}_s) \end{cases} \quad (28)$$

where $b > 0$ is a free design parameter.

We introduce now a modification to the original scheme proposed in [11] by stating the following definition:

$$\begin{cases} v_s &= \sqrt{1 - m_1}\; u_m(t - T_1(t)) \\ v_m &= \sqrt{1 - m_2}\; u_s(t - T_2(t)) \end{cases} \quad (29)$$

where m_1 and m_2 are the upper values of the time derivative of the delays, i.e. $m_1 \geq \dot{T}_1$ and $m_2 \geq \dot{T}_2$, respectively. Compared to the result in [11], we introduced an attenuation of the transmitted wave variables. This attenuation preserves the passivity of the system defined in the wave variables framework, that is guaranteed if

$$\int_0^t \frac{1}{2}\mathbf{v}^T\mathbf{v}dt \leq \int_0^t \frac{1}{2}\mathbf{u}^T\mathbf{u}dt \quad (30)$$

where $\mathbf{u}$ and $\mathbf{v}$ are the vectors of input and output wave variables, respectively.

If we send the attenuated wave variables, as shown in Figure 6, the power flow P results:

$$P = \frac{1}{2}u_m^2 - (1 - m_2)\frac{1}{2}u_s^2(t - T_2(t)) + \\ + \frac{1}{2}u_s^2 - (1 - m_1)\frac{1}{2}u_m^2(t - T_1(t)) \quad (31)$$

We now consider the energy provided to the communication system by the input wave variables:

$$E(t) = \int_{t-T_1(t)}^{t} \frac{1}{2}u_m^2(t)dt + \int_{t-T_2(t)}^{t} \frac{1}{2}u_s^2(t)dt \quad (32)$$

If we compute its time derivative, we obtain:

$$\frac{dE}{dt} = \frac{1}{2}u_m^2(t) - [\frac{1}{2}u_m^2(t - T_1(t))](1 - \dot{T}_1(t)) + \\ \frac{1}{2}u_s^2(t) - [\frac{1}{2}u_s^2(t - T_2(t))](1 - \dot{T}_2(t)) \quad (33)$$

since we have $m_1 \geq \dot{T}_1(t)$ and $m_2 \geq \dot{T}_2(t)$, this implies that for the system of Figure 6 it holds:

$$P \geq \frac{dE}{dt} \quad (34)$$

which is the condition for the overall passivity, since it is equivalent to condition (24). By knowing the upper value of the time derivative of the delay, it is then possible to find a decomposition of the power variables that ensures the overall stability of the telerobotic system. From the block diagram of Figure 6 it is easy to see that there has been a trade-off between stability and scaling in both velocity and force variables. Another remark is that, as a consequence of the results in (34), the communication channel is no longer lossless as in [11], but dissipative, since the istantaneous values of T_1 and T_2 cannot be known.

5 Experimental results

To prove the validity of the proposed solutions, an experimental setup was implemented. It consists of a force reflecting master and of a virtual slave, simulated in software, forming a virtual telemanipulator. The master is the 2-dof planar manipulator shown in Figure 7, with a 4 cm^2 working area, controlled by a laptop personal computer (PC) [2]. The slave is a virtual object modeled in the PC, controlled in position by the master. The master is connected to the PC via a PCMCIA I/O board and it communicates to the Internet using a UDP-based library [2] that satisfies the requirements for real-time Internet communication [8]. We show in Figure 8 the results of an experiment in

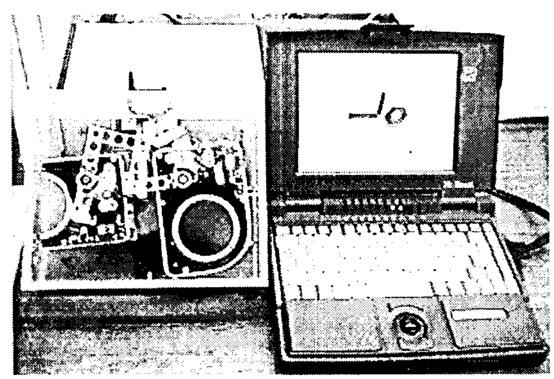

Figure 7: Telerobotic system used in the experiments

which the end effector of the virtual slave is pushed against a soft wall. The control scheme used here is the state feedback proposed in Section 3. More detailed description of the experimental system and results can be found in [13].

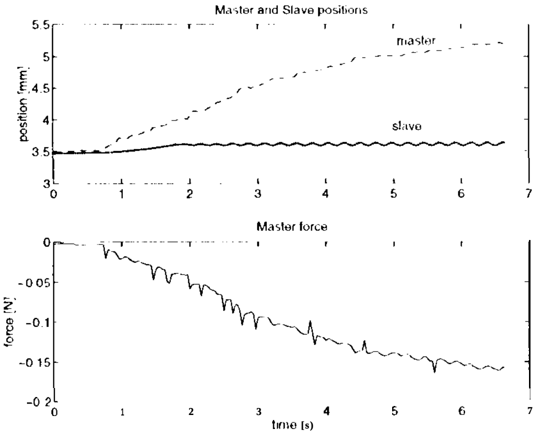

Figure 8: Pushing against a soft wall

6 Conclusions

We have presented in this paper two new methods for the stabilization of telerobots in two common configurations. This result has been obtained with the explicit inclusion of the variable delay characteristics. Preliminary experimental results (reported in [13]) have confirmed the validity of the proposed solution. For the further development of the research here summarized, it is worthnoticing that we have considered here only continuous-time systems. Master-slave communications, instead, are usually realized with a digital systems for which it is necessary to find a time continuous equivalent. It has been already shown in [13] that the digital communication system (e.g. Internet) has a time-continuous equivalent, but a more realistic approach requires the analysis of the problem in a discrete-time framework. This will be the subject of our future research.

7 Acknowledgement

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